Announcement

- Feedback for Project proposal latest tonight
- Given erroneous data provided for Q3, we extended the submission deadline till tonight
- Kevin Zakka started course notes (see Piazza) - bonus points for contributing
- No time after class today - CS300 Lecture at 4:30pm
What will you take home today?

Differentiable Filters
  Backpropagation through a Particle Filter

Introduction to Control
  PD Controllers
  PID Controllers
  Gain tuning
Differentiable Particle Filters: End-to-End Learning with Algorithmic Priors. Jonschkowski et al. RSS 2018.
Differentiable Particle Filters: End-to-End Learning with Algorithmic Priors. Jonschkowski et al. RSS 2018.
Differentiable Particle Filters: End-to-End Learning with Algorithmic Priors. Jonschkowski et al. RSS 2018.
Differentiable Particle Filters: End-to-End Learning with Algorithmic Priors. Jonschkowski et al. RSS 2018.

\[ e_t = h_\theta (o_t) \]
\[ S_t = h_\theta (e_t, \sum i \sim B) \]
\[ w_t = \lambda_\theta (e_t, S_t) \]
\[ \sim p(z | x_t) \]

**Measurement update**

- **Belief**
  - Resample
  - Insert particles
  - Set weights

- **Particles**
  - Move particles
  - Predicted poses
  - Predicted belief

- **Prediction**
  - Action \( A_t \)
  - Noisy actions
  - Dynamics model \( g/\theta \)
  - Action sampler \( f_\theta \)

**Particle flow**
- Predicted particles
- Observed encoder \( h_\theta \)
- Observ. likelihood estimator \( l_\theta \)
- Particle poses
- New particles

**Algorithmic Priors**
- does not depend on predicted particles

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\[ w^i_t = \theta(e_t, s^i_t) \quad \text{no dependence on previous weights} \]

\[ q(k) = \alpha w^k_t + (1-\alpha) \frac{1}{K} \]

\[ w^\prime_k = \frac{p(k)}{q(k)} = \frac{w^k_t}{\alpha w^k + (1-\alpha) \frac{1}{K}} \]

depends on the old weight through this junction.
Differentiable Particle Filter – Loss Function

\[ \theta^* = \text{argmin}_\theta - \log E_t[\text{bel}(s^*_t; \theta)]. \]
Differentiable Particle Filter – Experiments and Baselines

Fig. 5: **Learned motion model.** (a) shows predictions (cyan) of the state (red) from the previous state (black). (b) compares prediction uncertainty in x to true odometry noise (dotted line).

Fig. 6: **Learned measurement model.** Observations, corresponding model output, and true state (red). To remove clutter, the observation likelihood only shows above average states.
Differentiable Particle Filter – Experiments and Baselines

Fig. 9: Generalization between policies in maze 2. A: heuristic exploration policy, B: shortest path policy. Methods were trained using 1000 trajectories from A, B, or an equal mix of A and B, and then tested with policy A or B.
Differentiable Particle Filter – Experiments and Baselines

(a) Visual input (image and difference image) at time steps 100, 200, and 300 (indicated in (b) by black circles)

(b) Trajectory 9; starts at (0,0)

Fig. 10: Visual odometry with DPFs. Example test trajectory
What will you take home today?

Differentiable Filters
  Backpropagation through a Particle Filter

Introduction to Control
  PID Controllers
  Feedforward Controllers
Introduction to Control

\[ \mathcal{F}(r_t) = u_t \]
Open-Loop Control

- In case we have a good model of the robot
- \( F(r_t) = u_t \)

Disadvantages: no reaction to disturbances
- Model may be difficult to derive
  or its only approximate
Feedback Control

\[ e_t = (r_t - v_t) \]

The plant can check the result of its action. The goal is to drive this error to 0. The simplest possible controller is \( F(e_t) = k_p e_t \).

Proportional gain
Joint Space Control

- Springs at each joint with a certain stiffness
  ~ Proportional Gain

Setpoint in joint space

\[ q^i = \text{joint angles} \quad g^i_{\text{des}} = \text{setpoints} \]

\[ e = q^i - g^i_{\text{des}} \leq 0 \]
Task Space Control

CS 223a → Introduction to Robotics

$x_{\text{desired}}$
Joint Space Control

\[ q_d = f(x_d) \]

Inverse kinematics

\[ \delta q_d = \delta q_1 \rightarrow \text{Joint 1} \]
\[ \delta q_d = \delta q_2 \rightarrow \text{Joint 2} \]
\[ \delta q_d = \delta q_n \rightarrow \text{Joint n} \]
Task Space Control

\[ \tau = J^T F \]

Jacobian of the Forward Kinematics

desired
Joint Space - PD Controller

Propotional - Derivative Control Law in joint space

\[ \tau = -k_p (q - q_d) - k_D \dot{q} \]

Propotional Gain

\[ \rightarrow \text{Drivily} \]

\[ \rightarrow \text{Drive to 0} \]

Derivative Gain

\[ \rightarrow \text{Damps} \]

\[ \rightarrow \text{Stability} \]
Passive Natural Systems - Conservative

\[ \dot{x} = K - V = 0 \]
\[ \frac{\partial}{\partial t} \frac{\partial (K-V)}{\partial x} - \frac{\partial (K-V)}{\partial x} \]
\[ m \ddot{x} = F \]

\[ K = \frac{1}{2} m \dot{x}^2 \]
\[ V = \frac{1}{2} k x^2 \]

\[ m \ddot{x} = F = -kx \]
Passive Natural Systems - Conservative

\[ m \ddot{x} + kx = 0 \]

\[ \ddot{x} + \omega_n^2 x = 0 \]

Frequency of response increases with stiffness \( k \) and inverse mass

\[ \omega_n = \sqrt{\frac{k}{m}} \]

\[ V = \frac{1}{2} kx^2 \]
Passive Natural System – Dissipative

\[ \frac{\partial}{\partial t} \frac{\partial (k-v)}{\partial x} - \frac{\partial (k-v)}{\partial x} = \int \text{ friction} \]

Viscous friction: \( \int \text{ friction} = -bx \)

\[ m \ddot{x} + b \dot{x} + kx = 0 \]
Passive Natural System – Dissipative

\[ m\ddot{x} + b\dot{x} + kx = 0 \]

\[ \ddot{x} + \left( \frac{b}{m} \right) \dot{x} + \frac{k}{m} x = 0 \]

- Oscillatory damped
- Critically damped
- Over damped

Natural frequency:

\[ \frac{b}{m} = 2 \cdot \omega_n \]

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No Damping
Underdamped
Overdamped
Critically Damped
Critically Damped System – Choose B

\[ m\ddot{x} + b\dot{x} + kx = 0 \]

\[ \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0 \]

Natural damping ratio as a reference value

\[ \xi_n = \frac{b}{2\omega_n m} = \frac{b}{2\sqrt{km}} \]

Critically damped system: \( \xi_n = 1 \) \( (b = 2\sqrt{km}) \)
1 DOF Robot Control

Potential Field:
\[ V(x) > 0 \quad x \neq x_d \]
\[ V(x) = 0 \quad x = x_d \]
\[ V(x) = \frac{A}{2} k_p (x - x_d)^2 \]

\[ f = -\nabla V(x) = -\frac{\partial V}{\partial x} \]

\[ m\ddot{x} = -\frac{\partial}{\partial x} \left[ \frac{A}{2} k_p (x - x_d)^2 \right] \]
\[ m\ddot{x} + k_p (x - x_d) = 0 \]

Find a force that brings \( x \) to \( x_{des} \)

Potential energy

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Asymptotic Stability – Converging to a value

\[ \frac{d}{dt} \left( \frac{\partial (K - V)}{\partial x} \right) - \frac{\partial (K - V)}{\partial x} = -F_s \text{ dissipative} \]

\[ f^T x < 0 \quad \text{for} \quad x \neq 0 \]

\[ f = -k_d x \quad \text{with any} \quad k_d > 0 \]

Position gain

\[ f = -k_p (x - x_d) - k_d |x| \quad \text{Velocity gain} \]

\[ m \quad f \quad x_0 \quad x_d \]
Proportional Derivative Controller

\[ m\ddot{x} = f = -k_p(x - x_d) - k_v\dot{x} \]

\[ m\ddot{x} + k_p(x - x_d) + k_d\dot{x} = 0 \]
Test yourself
Control Partitioning
Non-Linearity

\[ f = \sum_{i=0}^{m-1} x_i \]

\[ f' = f + \hat{b}(x, \dot{x}) \]

\[ (x, \dot{x}) \]
Motion control

\[ f' = k_p'(x - x_d) + k_v' \dot{x} \]

\[ \ddot{x} + k_v' \dot{x} + k_p'(x - x_d) = 0 \]

\[ \ddot{e} + k_v' \dot{e} + k_p' e = 0 \]
Disturbance rejection

\[
m\dddot{x} + b(x, \dot{x}) = f + f_{\text{dist}}
\]
Steady-State Error

\[ \ddot{e} + k' \dot{e} + k' e = \frac{f_{\text{dist}}}{m} \]

The steady-state \( \dot{x} = \ddot{x} = 0 \)
Example
PID controller