What will we learn - Fundamentals of Image Formation

- Why are multiple views useful?
- Epipolar Constraints
- Essential and Fundamental Matrix
- Recovering Depth from Correspondences
Can you recover 3D structure from a single view?

Calibration rig

Scene

Camera K

O_w

P

p

Courtesy figure: Silvio Savarese.
Intrinsic ambiguity in the mapping from 3D to 2D
Two eyes help!

Courtesy figure: Silvio Savarese.
Two eyes help!

$P = l \times l'$

$K = \text{known}$

$K' = \text{known}$

$O_1$  $O_2$

$P$

$p$

$p'$

$l$

$l'$

$R, T$

Courtesy figure: Silvio Savarese.
Find $P^*$ that minimizes:

$$d(p, MP^*) + d(p', M'P^*)$$
Problems to solve given Multiview Geometry

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**Camera Calibration:** Given corresponding 2D projections of given 3D points in N images, find intrinsic and extrinsic camera parameters.

**3D Reconstruction:** Given corresponding projections in N images, estimate 3D coordinates of projected point

**Correspondence:** Given projected point p in one image, find the corresponding projection in an image taken from a different camera angle
Epipolar Geometry

Courtesy figure: Silvio Savarese.
Parallel Camera Coordinate Systems

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Courtesy figure: Silvio Savarese.
Exercise: Click on the Epipoles.
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- The epipoles have same position in both images
- Epipole called FOE (focus of expansion)

Courtesy figure: Silvio Savarese.
Epipolar Constraints help with finding correspondences

Where is $p'$?
Epipolar Constraints help with finding correspondences

Fig. 1: Epipolar Constraints. The epipolar lines for a point $p$ in the first image and $p'$ in the second image intersect at point $P$. This constraint helps in finding corresponding points between the two images. Courtesy figure: Silvio Savarese.
Epipolar Constraint

\[ M = K \begin{bmatrix} I & 0 \end{bmatrix} \]

\[ M \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p \end{bmatrix} \]  [Eq. 3]

\[ M' = K' \begin{bmatrix} R^T & -R^T T \end{bmatrix} \]

\[ M' \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} p' \end{bmatrix} \]  [Eq. 4]
Epipolar Constraint

\[ p' \text{ in first camera reference system is } = R \ p' + T \]
\[ T \times ((R \ p') + T) = T \times (R \ p') \] is perpendicular to epipolar plane

\[ \rightarrow p^T \cdot [T \times (R \ p')] = 0 \] [Eq. 7]
Cross Product as Matrix Multiplication

\[
\mathbf{a} \times \mathbf{b} = \begin{bmatrix}
0 & -a_z & a_y \\
a_z & 0 & -a_x \\
-a_y & a_x & 0 \\
\end{bmatrix}
\begin{bmatrix}
b_x \\
b_y \\
b_z \\
\end{bmatrix} = [\mathbf{a}_\times] \mathbf{b}
\]
Epipolar constraint

\[ p^T \cdot [T \times (R \ p')] = 0 \rightarrow p^T \cdot [T \times] \cdot R \ p' = 0 \]

[Eq. 8] [Eq. 9]

E = Essential matrix
(Longuet-Higgins, 1981)

Courtesy figure: Silvio Savarese.
Epipolar constraint

\[ p^T \cdot E \cdot p' = 0 \]  

[Eq. 10]
Adding back Intrinsic Camera Parameters

\[ F = K^{-T} \cdot [T_x] \cdot R \; K'^{-1} \]

**F = Fundamental Matrix**

(Faugeras and Luong, 1992)

\[ p^T F p' = 0 \]  \hspace{1cm}  \text{[Eq. 13]}

\[ M = K \begin{bmatrix} I & 0 \end{bmatrix} \]

\[ M' = K' \begin{bmatrix} R^T & -R^T \; T \end{bmatrix} \]

\[ M \; P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = p \]  \hspace{1cm}  \text{[Eq. 3]}

\[ M' \; P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = p' \]  \hspace{1cm}  \text{[Eq. 4]}

Courtesy figure: Silvio Savarese.
Why is the Fundamental Matrix $F$ useful?

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Given a point in the left image, we can compute the Epipolar line in the second image. Only search for correspondences along this line without further camera info.

Gives constraints on how the scene changes under viewpoint changes (without reconstruction)
Estimating the Fundamental Matrix - 8 Point Algorithm

Courtesy figure: Silvio Savarese.
Summary - Essential Matrix

\( p^T \mathbf{E} p' = 0 \)

\[ \mathbf{E} = \begin{bmatrix} T_x \end{bmatrix} \cdot \mathbf{R} \]

\[
\mathbf{E} = \begin{bmatrix}
0 & -T_z & T_y \\
T_z & 0 & -T_x \\
-T_y & T_x & 0
\end{bmatrix}
\]

Courtesy figure: Silvio Savarese.
Example - Parallel Planes

K₁ = K₂ = known
x parallel to O₁O₂

E = ?

Courtesy figure: Silvio Savarese.
Depth from Correspondences

\[ \text{disparity} = p_u - p'_u \propto \frac{B \cdot f}{z} \]

Disparity is inversely proportional to depth \( z \)!

Courtesy figure: Silvio Savarese.
Computing Depth

\[ \text{disparity} = p_u - p'_u \propto \frac{B \cdot f}{z} \]  

[Eq. 1]

Courtesy figure: Silvio Savarese.
\[ p_u - p'_u \propto \frac{B \cdot f}{z} \]  

[Eq. 1]

Stereo pair
Correspondence Problem

\[ p \text{ belongs to } l = F p' \]

\[ p' \text{ belongs to } l' = F^t p \]
Finding Correspondences - Correlation Method

\[ \bar{p} = \begin{bmatrix} \bar{u} \\ \bar{v} \\ 1 \end{bmatrix} \quad \bar{p}' = \begin{bmatrix} \bar{u}' \\ \bar{v} \\ 1 \end{bmatrix} \]
Questions?